International Journal of Forecasting 3 (1987) 171–177
North-Holland

TIME-VARYING RISK PREMIA, IMPERFECT INFORMATION AND THE FORWARD EXCHANGE RATE

Rene M. STULZ *

The Ohio State University, Columbus, OH 43210, USA

Abstract: The evidence for the hypothesis that forward exchange rates incorporate time-varying risk premia implies that risk premia are highly volatile and that their sign changes often. I argue that existing models of asset pricing are unlikely to yield risk premia whose sign changes often. I show that a model in which individuals learn over time about exchange rate dynamics yields forecasting errors for the forward exchange rate whose time-series properties would be suggestive of a time-varying risk premium even in the absence of a risk premium.

Keywords: Forward exchange rate, Risk premium, Imperfect information.

1. Introduction

Since the demise of the fixed exchange rate system, numerous papers have been written on the issue of the efficiency of the market for foreign exchange. Initially, researchers believed that a key implication of foreign exchange market efficiency is the hypothesis that the forward exchange rate is an unbiased predictor of the future spot exchange rate. As empirical procedures increased in sophistication and as more data became available, evidence against this hypothesis was found to be overwhelming and led to the recognition of the importance of risk premia in the determination of forward exchange rates [1].

Most of the research which rejects the hypothesis that the forward exchange rate is an unbiased predictor of the future spot exchange rate does so in a subtle way. When long periods of time are used, it is generally not possible to show that the forward exchange rate either systematically *under*-predicts or systematically *over*-predicts the future spot exchange rate. In fact, many papers show that, over long periods of time, the average prediction error of forward exchange rates is surprisingly close to zero. [2] However, in general, successive prediction errors of forward exchange rates are correlated. [3] This means that, in principle, knowledge of the past forecasting errors of the forward exchange rate for a particular currency enables one to form a forecast for the future spot exchange rate which is a better forecast than the forward exchange rate itself.

* I am grateful for comments from Don Alexander, Jim Bodurtha, K.C. Chan, Bob Cumby, Nelson Mark and from an anonymous referee.

[1] For a review of the early research, see Levich (1979). Cumby and Obstfeld (1984) provide a discussion of the more recent research.

[2] See, for instance, the evidence in Hansen and Hodrick (1980) and Cumby and Obstfeld (1984). These authors regress percentage forward exchange rate forecast errors on a set of information variables and generally find that the constant term of such equations is not significant.

Researchers generally interpret the evidence that successive forecasting errors of forward exchange rates are correlated to be evidence in favor of the hypothesis that forward exchange rates incorporate time-varying risk premia. This is because, if market participants know the true distribution of exchange rate changes, market efficiency implies that, in the absence of risk premia, forward exchange rates are unbiased predictors of future spot exchange rates. For time-varying risk premia alone to explain the empirical evidence, their sign has to change frequently enough to yield average forecasting errors close to zero. In this paper, I argue that existing models of asset pricing are unlikely to yield time-varying risk premia whose sign changes often enough to explain all the empirical evidence. If one believes that the hypothesis of time-varying risk premia cannot explain the empirical evidence completely, one is not forced to conclude that the market for foreign exchange is inefficient and offers numerous opportunities for risk-adjusted profits. I show that it is reasonable to expect successive forecasting errors of forward exchange rates to be correlated when market participants learn about the true exchange rate dynamics over time. In this case, serial correlation in forecasting errors does not imply that foreign exchange markets are inefficient, as it is still possible for all available information to be incorporated in forward exchange rates.

This paper is organized as follows. In section 2, I discuss the implications of existing asset pricing models for the pricing of forward contracts for foreign exchange. In section 3, I present a simple model in which households learn about exchange rate dynamics over time and show that this model has useful empirical implications. Finally, in section 4, I provide some concluding remarks.

2. Asset pricing models and time-varying risk premia

In this section, I review the implications of asset pricing models for the pricing of forward contracts for foreign exchange. To simplify, I focus the discussion on a model which incorporates most other models as special cases. This model, discussed extensively in Stulz (1981), assumes that trading takes place continuously, that markets are internationally perfect (i.e., there are no taxes, no transactions costs, and individual traders cannot affect prices), that individuals maximize lifetime expected utility functions which are state-independent and time-separable, and that all exogenous variables follow jointly a Markov diffusion process. [4] Such assumptions are fairly standard in the asset pricing literature and I will show later that relaxing most of these assumptions would not affect my argument.

In an economy which satisfies the assumptions just described, the forward premium on a contract of instantaneous maturity satisfies

$$\mu_{\mathrm{f}} = \mu_e - \sigma_{e,P} - \beta_{ec}[\mu_c - r], \tag{1}$$

where μ_{f} is the forward premium, μ_e is the expected rate of change of the spot exchange rate e (defined as the domestic currency price of one unit of foreign currency), $\sigma_{e,P}$ is the instantaneous covariance between the rate of change of the spot exchange rate and the rate of change of the

[3] See, for instance, Cumby and Obstfeld (1981, 1984) and Hansen and Hodrick (1983). It is important to note that when forecast errors are regressed on a constant and lagged forecast errors, the regression coefficient for each of the lagged forecast errors is usually not significantly different from zero at conventional confidence levels. However, generally, the hypothesis that the regression coefficients of all lagged forecast errors are jointly zero can be rejected strongly. Fama (1984) provides useful evidence about the variability of the risk premium under the assumption that individuals know the true distribution of exchange rate changes.

[4] Stulz (1984) shows how this model incorporates alternative models as special cases. There is some evidence that foreign exchange changes do not follow a diffusion process; see Hsieh (9185) and the references therein.

domestic price level, β_{ec} is the instantaneous consumption beta of the exchange rate, i.e., the instantaneous covariance of the rate of growth of e with the rate of growth of real consumption divided by the variance of the rate of growth of real consumption, μ_c is the expected real rate of return on a portfolio whose real return is perfectly correlated with the growth rate of real consumption and, finally, r is the expected real rate of return on an asset whose return is uncorrelated with the growth rate of real consumption. Eq. (1) implies that the risk premium incorporated in the forward exchange rate, $\beta_{ec}[\mu_c - r]$, is an increasing function of the consumption beta of the spot exchange rate, as $\mu_c - r$ must always be positive. The covariance term $\sigma_{e,P}$ implies that the forward exchange rate can be a biased predictor of the future spot exchange rate even in the absence of a risk premium. This term, caused by the fact that the real payoff of a forward contract is a convex function of the price level, plays no role in our discussion. We therefore proceed as if $\sigma_{e,P}$ is negligible. [5]

Eq. (1) follows from the fact that, everything else equal, individuals prefer assets which have high real payoffs when their real consumption is low to assets which have high real payoffs when their real consumption is high. This is because assets which have high real payoffs when real consumption is low enable individuals to smooth out their consumption over time and rational risk-averse individuals wish to do so. The payoff of a long position in a forward contract increases with the spot exchange rate. Hence, if e covaries positively with real consumption, a long position in a forward contract has a high payoff when real consumption is high, which means that households require a risk premium to take such a position, i.e., they want a higher expected real payoff than for the case in which the instantaneous covariance of the rate of growth of e with real consumption growth is zero or negative. As the exchange rate dynamics and the current spot exchange rate are taken as given, an increase in the expected payoff of a forward contract can only be obtained by a fall in the forward exchange rate. Hence, the forward exchange rate must be a decreasing function of the covariance of the rate of change of the spot exchange rate with consumption growth.

It immediately follows from eq. (1) that the sign of the risk premium incorporated in the forward exchange rate can change only because the sign of the instantaneous covariance between the rate of change of e and the growth rate of consumption changes. To understand why it would be surprising if the sign of the consumption beta of a currency changed often, it is useful to notice that the consumption beta of a currency can be rewritten as a weighted sum of the consumption betas of the exogenous variables which drive exchange rate changes. [6] The spot exchange rate can be represented as a twice differentiable function of a vector of S state variables, S, so that we can write $e = e(S)$. Using this notation, the consumption beta of the spot exchange rate can be rewritten as

$$\beta_{ec} = \sum_{i=1}^{S} \eta_{S_i} \beta_{S_i c}, \tag{2}$$

where η_{S_i} is the elasticity of the spot exchange rate with respect to the ith state variable and $\beta_{S_i c}$ is the consumption beta of the ith state variable. [7] Eq. (2) implies that the consumption beta of a

[5] For evidence that this is not an unreasonable assumption, see Cumby (1986).

[6] There is some evidence that sample betas exhibit substantial volatility; see Cosset (1984) for capital asset pricing model betas and Çumby (1986) for consumption betas. it is important to note, however, that the theory requires the use of expected betas. Domowitz and Hakkio (9185) provide direct estimates of the risk premium using a version of the model of Lucas (1982). They show that the sign of the risk premium changes often. However, the risk premium they estimate correspond to our $\sigma_{e,P}$ term and hence may be different from zero even if households are risk neutral.

[7] To obtain eq. (2), note that, using Ito's Lemma, $\mathrm{d}e = \Sigma_{i=1}^{S} \partial e/\partial S_i \mathrm{d}S_i + \ldots$, where the omitted terms are non-stochastic. Hence, $\mathrm{d}e/e = \Sigma_{i=1}^{S} \partial e/\partial S_i S_i/e \; \mathrm{d}S_i/S_i + \ldots$, and eq. (2) follows by multiplying this expression on both sides by $\mathrm{d}c/c$.

currency can change sign only because the weighted sum of the consumption betas of the exogenous variables which drive the spot exchange rate changes sign.

To understand how the consumption beta of the exchange rate can change sign, suppose first that the η_S's are constant. In this case, the consumption beta of the exchange rate can change sign only through large changes in the consumption betas of the state variables which drive the exchange rate. Note, however, that not all changes in the consumption betas of these state variables lead to a change in the sign of the consumption beta of the spot exchange rate, as some large changes in the consumption betas of state variables can simply increase the absolute value of the consumption beta of the spot exchange rate. Hence, for the sign of the consumption beta of the exchange rate to change often, it is necessary that the consumption betas of the state variables exhibit considerable volatility. In most models of exchange rate determination, domestic and foreign outputs, money supplies and interest rates play a key role. It seems unlikely that the covariance of these variables with the growth rate of real consumption is very volatile or changes sign often. To wit, a change in the sign of the consumption beta of the money supply means, for instance, that monetary policy becomes pro-cyclical after having been counter-cyclical. While such changes in monetary policy sometimes take place, they are not frequent and are widely noticed whenever they take place.

If changes in the sign of the consumption beta of the spot exchange rate do not come about because of large changes in the consumption betas of the variables which drive exchange rates, they could be caused by changes in the elasticity of the spot exchange rate with respect to these variables. For instance, if the consumption beta of the spot exchange rate is negative while the consumption beta of the domestic money supply is positive, a large increase in the elasticity of the spot exchange rate with respect to the domestic money supply could change the sign of the consumption beta of the spot exchange rate. There are, however, obvious limits to changes in these elasticities. For instance, most economists would be extremely surprised if the elasticity of the spot exchange rate with respect to the money supply was found to be negative or extremely large. While the regression coefficients of exchange rate equations can change substantially over time, they rarely change by large amounts over short periods of time. [8] It would seem therefore that the relation between the spot exchange rate and the variables on which it depends is too stable to account for frequent changes in the sign of the risk premium unless the risk premium is small in absolute value. However, if the risk premium is close to zero, it cannot account for the observed serial correlation in the forecasting errors of the forward exchange rate.

One could argue that there is a time-varying risk premium which changes a lot over time, but that the asset pricing model used in this section neglects some important variables which affect the risk premium. This argument, however, seems unlikely to be valid, as the model used here incorporates most other models as special cases. The asset pricing relation does no longer hold if one relaxes the assumption of a state-independent time-separable utility function. [9] However, it is doubtful that removing this assumption would dramatically change one's perception of the risk of foreign currencies. Barriers to international investment could introduce more variability in risk premia, as they would make it harder for risks to be shared internationally and hence would increase the importance of the distribution of the wealth of individual countries in asset pricing relations. However, it follows from Stulz (1984) that with barriers to international investment which do not change much over time, the sign of the risk premium changes only when the consumption beta of the spot exchange rate changes, so that the arguments of this section still hold. If one changes the market structure, then the asset pricing relation given in eq. (1) can no longer be derived in the same way. A

[8] Alexander and Thomas (1986) provide indirect evidence on this as they show that allowing for time-varying parameters in exchange rate equations improves the performance of these equations only in a limited way.

[9] See Bergman (1985).

better understanding of the microstructure of markets for common stocks has enabled financial economists to better understand the time-series properties of daily stock returns. However, it is unclear that much would be gained by a better understanding of the microstructure of the market for foreign exchange. The serial correlation of the forecasting errors of forward exchange rates holds on weekly data [see Cumby and Obstfeld (1981)], for example, which could not be explained by institutional details of how prices are set and reported.

3. A learning model

Traditionally, researchers assume that investors know the true distribution of exchange rate changes. Given the rather shaky state of our understanding of exchange rate dynamics, such an assumption seems excessive. Yet, this assumption plays a large role in tests of the existence of time-varying risk premia, as it implies that any persistence in forecasting errors of the forward exchange rate can be attributed to a risk premium if the market for foreign exchange is efficient. Without this assumption, persistence in forecasting errors can occur because investors learn over time about the true distribution of exchange rate changes.

To make my point, I use a very simple model in which individuals learn over time about exchange rate dynamics. [10] Consider an economy in which the true distribution of the growth rate of the spot exchange rate is unknown. However, all individuals know that the exchange rate follows a lognormal diffusion process

$$\frac{\mathrm{d}e}{e} = \mu_e \,\mathrm{d}t + \sigma_e \,\mathrm{d}z_e, \tag{3}$$

where $\mathrm{d}z_e$ is the increment of a standard Wiener process. It is well-known in the continuous-time finance literature that if a price follows eq. (3) and individuals do not know μ_e and σ_e, they can observe σ_e immediately but learn about μ_e over time. [11] At any point in time, individuals form a predictive distribution for the growth rate of the spot exchange rate using all the information available to them. For simplicity, we focus the discussion on the case in which the only relevant information available to individuals is the time series of past changes of the spot exchange rate. In this case, the individuals' predictive distribution over the growth rate of the spot exchange rate has mean $\hat{\mu}_e(t)$ and variance $\hat{\sigma}_e^2(t)$ given by

$$\hat{\mu}_e(t) = \tfrac{1}{2}\sigma_e^2 + \frac{1}{t}\ln\frac{e(t)}{e(0)}, \tag{4}$$

$$\hat{\sigma}_e^2(t) = \left(\frac{t+1}{t}\right)\sigma_e^2, \tag{5}$$

where t is the time elapsed since the exchange rate started to follow eq. (3). It is important to notice that $\hat{\mu}_e(t)$ can exceed μ_e or be lower than μ_e for significant periods of time. To see this, suppose that the growth rate of the spot exchange rate exceeds its mean for some period of time. This leads to a high value of $\hat{\mu}_e(t)$ compared to μ_e and hence, for a while, individuals expect too high of an increase in the spot exchange rate.

[10] See Stulz (1986) for a general equilibrium model which yields results similar to those presented here. Stulz (1986) also discusses how such models could be generalized.

[11] See Williams (1977).

The asset pricing model used in section 2 still applies when investors do not know the true expected growth rate of the spot exchange rate, except that the true distribution of asset returns and consumption growth, if unknown, is replaced by the predictive distribution. To simplify the discussion, however, we consider the special case such that a forward contract is not risky, in the sense that the consumption beta of the spot exchange rate is equal to zero. In this case, using eq. (1), the forward premium at date t is given by

$$f(t)=\hat{\mu}_e(t)-\hat{\sigma}_{e,P}(t), \tag{6}$$

where $\hat{\sigma}_{e,P}(t)$ is the instantaneous covariance of the growth rate of the spot exchange rate with the rate of inflation at home given the individuals' predictive distribution over the growth rate of the exchange rate. As previously, the covariance term in the equation for the forward premium is assumed to be negligible. Even though the true expected rate of growth of the spot exchange rate is constant and the consumption beta of the spot exchange rate is equal to zero, the forward exchange rate differs from the true expected spot exchange rate. This can best be seen by rewriting the forward premium as

$$f(t)=\hat{\mu}_e(t)-\hat{\sigma}_{e,P}(t)=\mu_e+\left(\hat{\mu}_e(t)-\mu_e\right)-\hat{\sigma}_{e,P}(t). \tag{7}$$

In this case, the term usually identified as the risk premium is simply the difference between the individuals' estimate of the expected rate of change of the spot exchange rate and the true expected rate of change of the spot exchange rate. Over long periods of time, the average value of $\hat{\mu}_e(t)$ is likely to be close to μ_e. However, $\hat{\mu}_e$ oscillates slowly around μ_e, giving the impression of a risk premium whose sign changes often. Hence, this very simple model yields some results which are qualitatively similar to those observed empirically. A major limitation of our simple model is that, because the true distribution of the rate of growth of the spot exchange rate is constant, households' estimate of the true mean growth rate of the exchange rate grows in precision with time. In fact, as t goes to infinity, the households learn the exact value of the true mean growth rate of the exchange rate, so that, if $\hat{\sigma}_{e,P}(t)$ is negligible for all t, eq. (7) becomes such that $f(\infty)=\mu_e$. Hence, in this model, the absolute value of the perceived risk premium would, on average, fall over time, which would seem to contradict the empirical evidence. However, a more complete model would allow for changes in the true mean growth rate of the exchange rate, i.e., it would let μ_e depend on a vector of state variable.[12] In this case, households would have to learn about a variable which changes stochastically over time and hence the precision of their estimate of μ_e would increase only when μ_e does not change much over a period of time.

4. Concluding remarks

In this paper, I have argued that the hypothesis of time-varying risk premia is unlikely to explain all of the existing empirical evidence about the persistence of the forecasting errors of forward exchange rates. While existing models of asset pricing offer strong arguments for the existence of time-varying risk premia, they do not suggest that the sign of the risk premia changes often. The persistence in forecasting errors is more likely to be explained by the fact that individuals never know the true distribution of changes in the spot exchange rate, but learn about that distribution in periods when it does not change too much. This, of course, does not mean that forward exchange rates do

[12] Lipster and Shiryaev (1977) derive techniques which can be used in this general case.

not incorporate risk premia. It is important, however, to remember that financial economists have always had trouble to find direct evidence of risk premia when examining the returns of individuals assets as opposed to portfolios of assets. This is because asset prices are generally too volatile to make it possible to find significant differences in mean returns. As forward contracts are similar to individual assets, one should therefore not be surprised to find it difficult to uncover risk premia directly. [13]

References

Alexander, D. and L.R. Thomas, III, 1986. Monetary asset models of exchange rate determination: How well have they performed in the 1980's?, This issue.

Bergman, Y.Z., 1985, Time preference and capital asset pricing models, Journal of Financial Economics 14, 145–160.

Cosset, J.C., 1984, On the presence of risk premiums in foreign exchange markets, Journal of International Economics 16, 139–154.

Cumby, R.E., 1986, Is it risk? Explaining deviations from uncovered interest parity, Unpublished manuscript (New York University, New York).

Cumby, R.E. and M. Obstfeld, 1981, A note on exchange-rate expectations and nominal interest differentials: A test of the Fisher hypothesis, Journal of Finance 36, 697–704.

Cumby, R.E. and M. Obstfeld, 1984. International interest rate and price level linkages under flexible exchange rates: A review of recent evidence, in: J.F.O. Bilson and R.C. Marston, eds., Exchange rate theory and practice (University of Chicago Press, Chicago, IL).

Domowitz, I. and C.S. Hakkio, 1985, Conditional variance and the risk premium in the foreign exchange market, Journal of International Economics 19, 47–66.

Fama, E.F., 1974. Forward and spot exchange rates, Journal of Monetary Economics 14, 319–338.

Hansen, L.P. and R.J. Hodrick, 1980, Forward exchange rates as optimal predictors of future spot rates: An econometric analysis, Journal of Political Economy 88, 829–853.

Hansen, L.P. and R.J. Hodrick, 1983, Risk averse speculation in the forward exchange market: An econometric analysis of linear models, in: J.A. Frenkel, ed., Exchange rates and international macroeconomics (University of Chicago Press, Chicago, IL).

Hsieh, D., 1985, The statistical properties of daily foreign exchange rates: 1979–1983, Unpublished paper (University of Chicago, Chicago, IL).

Levich, R.M., 1979, On the efficiency of markets for foreign exchange, in: R. Dornbusch and J.A. Frenkel, eds., International economic policy: Theory and evidence (Johns Hopkins University Press, Baltimore, MD).

Lipster, R.S. and A.N. Shiryaev, 1977, Statistics of random processes, Vol. I and II (Springer Verlag, New York).

Lucas, R.E., Jr., 1982, Interest rates in a two-country world, Journal of Monetary Economics 10, 335–360.

Mark, N.C., 1985, On time-varying risk premia in the foreign exchange market: An econometric analysis, Journal of Monetary Economics 16, 3–18.

Stulz, R.M., 1981, A model of international asset pricing, Journal of Financial Economics 9, 383–406.

Stulz, R.M., 1984, Pricing capital assets in an international setting: An introduction, Journal of International Business Studies, 55–73.

Stulz, R.M., 1986, An equilibrium model of exchange rate determination and asset pricing with non-traded goods and imperfect information, Unpublished manuscript.

Williams, J.T., 1977, Capital asset prices with heterogeneous beliefs, Journal of Financial Economics 5, 219–240.

Biography: Rene STULZ is Visiting Professor of Finance at the University of Chicago and Professor of Finance at The Ohio State University. He has also taught at the University of Rochester and at the Massachusetts Institute of Technology. He has published in the fields of finance, macroeconomics and international economics. He is an editor of the *Journal of Financial Economics* and has served as associate editor of the *Journal of Financial and Quantitative Analysis.*.

[13] Marks (1985) provides a direct test on individual currencies of a model analogous to the one described in section 2 and provides only weak evidence against it. This suggests that there is some hope for tests of asset pricing models on suitably chosen portfolios of currencies. However, Cumby (1986), using a latent variable approach, provides stronger evidence against the same model.