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Discounting by Probabilistic Waiting

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ABSTRACT

In everyday life, many probabilistic situations may be characterized as probabilistic waiting. A gambler, for example, bets repeatedly at the racetrack, the casino, or the card table. The gambler may not win on the first try, but if a gamble is repeated enough times, a win is almost certain to occur eventually. If repeated gambles are structured as strings of losses ending in a win (probabilistic waiting) and the amount won is discounted by the delay caused by the series of losses, then strings with many losses will be discounted more than those with fewer losses, thereby causing subjective value of the series of gambles as a whole to increase. The current study used the opposite effect that amount has on the degree of delay and probability discounting as a marker to determine whether people evaluate situations involving probabilistic waiting as they evaluate situations involving delayed outcomes or as situations involving probabilistic outcomes. We find that the more likely a probabilistic waiting situation is to end in reward (e.g., a gamble is repeated indefinitely until reward is obtained), the more that situation conforms to delay discounting; the less likely a probabilistic waiting situation is to end in reward (e.g., a fixed, small number of gambles), the more that situation conforms to probability discounting. We argue that the former situation is applicable to pathological gambling, and that people with steep delay discount functions would therefore be more likely to have gambling problems. Copyright © 2015 John Wiley & Sons, Ltd.

KEY WORDS amount effect; delay discounting; gambling; probability discounting; repeated gambles

Suppose that where you live, in this season, the probability of a day being a sunny day is \( p \), and that you are planning to take a pleasant drive to the country on the next sunny day. As \( p \) decreases, you should expect to wait longer to take that drive, and therefore, its present, current value should be less —that is, the current value of going for a drive on the next sunny day will be discounted more as \( p \) decreases. Any situation, such as the one earlier, where the chance of something happening is repeated until the event actually happens, we call probabilistic waiting. Situations like this, unlike one-shot gamble situations, involve both a probability component (e.g., the probability of a sunny day, \( p \)) and a delay component (e.g., the number of days until a sunny day occurs). The question then is whether choice in a probabilistic waiting situation is better understood primarily as a function of the delay or primarily as a function of the probability.

Choices involving delayed outcomes and choices involving probabilistic outcomes have been separately studied and understood within the discounting framework. Delay discounting refers to the devaluation of an outcome as the delay until its receipt increases, and probability discounting refers to the devaluation of an outcome as the odds against its receipt increases. Delay and probability discounting are both well described by a hyperboloid function (Green & Myerson, 2004) when choosing between an immediate-certain reward and a delayed or a probabilistic reward:

\[ V = \frac{A}{(1 + bX)^s} \]

where \( V \) is the amount of an immediate-certain reward equal in subjective value to the delayed or the probabilistic amount, \( A \); \( X \) is either the delay to \( A \) (termed \( D \)) or the odds against receiving \( A \) (termed \( \theta \), where \( \theta = (1 - p)/p \); and \( b \) and \( s \) are free parameters that may differ depending on the type of reward, context of reward, and characteristics and history of the organism making the choice (Green, Myerson, & Ostaszewski, 1999b; Green, Myerson, & Vanderveldt, 2014; Odum, Baumann, & Rimington, 2006). The expected waiting time for a probabilistic reward over a series of chances, assuming for the moment no waiting time to the first chance, equals \( \theta t \), where \( t \) is the interval between repetitions.1

To the extent that a single equation (Equation (1)) describes the discounting of a delayed reward and the discounting of a probabilistic reward, the two types of discounting would appear to correspond and may suggest that similar processes underlie both types of discounting. However, the relation between the free parameters \( (b \) and \( s \) and the undiscounted amount \( A \) in Equation (1) differs depending on whether \( X \) in Equation (1) represents the odds against \( \theta \) a one-shot probability of an immediate reward or represents the fixed delay \( D \) to a certain reward (Green, Myerson, Oliveira, & Chang, 2013; Myerson, Green, & Morris, 2011). The discounting of probabilistic and of delayed rewards by humans may be clearly distinguished by the opposite direction that amount of reward has on the degree of discounting in the two cases. An individual reward of a smaller amount is discounted more steeply by delaying it (termed the amount effect), and less steeply by decreasing its

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1This formula uses a fixed interval for the delay. Mazur (1984, 1986, 1989) has argued for the use of a distribution of delays to estimate subjective value of rewards with variable intervals between rewards. For the purposes of the current experiment, whether a fixed or variable interval is used to estimate expected delay is irrelevant, as our argument remains the same.
probability (termed the reverse amount effect), than is a reward of a larger amount. This difference in the effect of amount on degree of discounting is a highly reliable finding (Green & Myerson, 2004; Green, Myerson, & Ostszewski, 1999a). The purpose of the present set of experiments was to determine the effect that amount has on discounting in the case of probabilistic waiting, which has characteristics of both delay and probability.

A setting where probabilistic waiting is common is the gambling casino where players typically bet repeatedly. It may seem that subjective evaluation of a gamble is a paradigmatic case of probability discounting (Holt, Green, & Myerson, 2003; Madden, Petry, & Johnson, 2009; Petry, 2012). However, the problem with using probability discounting to examine gambling is that discounting studies typically are based on one-shot gambles, and gamblers frequently bet repeatedly in situations like casinos, racetracks, and card games. Rachlin’s “string theory” of repeated gambling (Rachlin, 1990; Rachlin et al., 2015; Rachlin, Siegel, & Cross, 1994) claims that the value of repeated gambles depends on structuring them as a string of losses followed by a win, and the overall amount won or lost on the string is then discounted by the delay to the win (as measured by the number of losses preceding it). Finally, across all possible strings, the discounted values are summed. In a series of even-money gambles (e.g., coin flips; L represents a loss and W represents a win), the most likely string is an immediate win (W), the next most likely is a loss followed by a win (LW), and so forth (Rachlin et al., 2015). If gamblers discount such strings by the delay to a win, they will discount strings with many losses more than strings with fewer losses, and overall losses will be discounted proportionally more than overall wins. Thus, the subjective value of the series as a whole will increase.\footnote{The term “gamble” can be used in different ways. The term has been used to mean a single probabilistic event (e.g., a coin flip) and also has been used to represent an infinite number of these events, for which there is an expected, long-term rate of return value. For clarity, we use the term “gamble” to mean a single probabilistic event, “string” to mean a series of gambles that ends in a win, and “series of strings” to mean a group of several strings. For example, LLW/LW is a series of two strings (an LLW string and an LW string), and each string is composed of three and two gambles, respectively.}

Table 1 shows how the expected and subjective values are calculated for the 10 most probable strings of an even-money gamble in which one bets $1 for the chance to win $2 on each gamble. Note that the positively valued string (W) is the most probable and the least delayed. Adding up the expected values of the (infinite number of) possible strings of losses preceding a win, the series of strings as a whole has an expected value of 0; a gambler would eventually end up neither winning nor losing any money. Suppose, however, that the subjective value of the series of strings is not equal to the expected value; rather, the expected value of each string is discounted by its delay (length of string until a win), and the sum of these discounted values is the subjective value of the series of strings. In Table 1, discounting is assumed to be hyperbolic according to Equation (1), where $V$ is the subjective value of an individual string, $A$ is the expected value of the string, $X$ is the delay in terms of number of losses in the string, and, for the sake of simplicity, the parameters $b$ and $s$ are assumed to equal 1.0. As Table 1 shows, the sum of the subjective values of the 10 most likely strings as a whole is positive (+.387) despite its near-zero expected value.

Even gambles with negative expected values may have positive subjective values according to string theory. That is, although the sum of all possible undiscounted strings of a zero expected-value gamble must be zero, the sum of all possible discounted strings of a zero expected-value gamble will be positive and will become more positive as the strings are discounted more steeply.\footnote{There is an exception to this. As rate of delay discounting becomes steeper and steeper, eventually even the very short positive strings will be highly discounted, and the overall subjective value of the gamble will decrease, approaching the value of an immediate win. However, Rachlin et al. (2015) showed that such high rates of delay discounting are well out of the range of obtained discount rates—even those of severe addicts.} The reason for this is that because they involve longer delays, individual long strings of losses followed by a win (net losses) are discounted more steeply than individual short strings of losses followed by a win (net wins). The strings in Table 1 are a case in point. The steeper the discounting, the less losses subtract, and the greater the subjective value of the string. Thus, the more steeply a person discounts delayed rewards (the greater is $b$), the more that person would value a repeated gamble, and the more likely it is that person would have a gambling problem. String theory proposes that a repeated gamble is subjectively discounted by delay until a win rather than by the probability of each individual gamble, and that the steeper the delay discounting, the more subjectively valuable would be the string of gambles, and therefore, the more tempting it would be to gamble. Some support for this model is found in the fact that pathological gamblers tend to discount delayed monetary rewards more steeply than do non-gamblers (Petry & Madden, 2010; Wiehler & Peters, 2015).

The present set of experiments directly tests whether probabilistic waiting is best understood as fundamentally due to probability or delay by varying the probability of a series of repeated gambles and determining whether the degree of discounting increases or decreases with amount, as is the case with probability discounting and delay discounting, respectively. Note that the current study examines the probability of winning without costs. Although string theory and gambling studies typically consider both winning and losing, the current experiments focus on the probability of winning in order to simplify this initial empirical attempt to examine gambling from the string theory framework. Previous studies have found meaningful distinctions between gamblers and non-gamblers using delay discounting and probability discounting of rewards (for a review, see Wiehler & Peters, 2015); therefore, by using the opposite amount effects of delay and probability reward discounting as a marker, we can determine how people evaluate probabilistic waiting.

The first experiment determined the amount of an immediate, certain reward that was equivalent in subjective value to a fixed probabilistic reward repeated each week until the participant won. The second experiment was similar to the first except that the interval between gambles was varied. The third
experiment extended the findings of the first two experiments to situations involving a fixed number of attempts to win the gamble, rather than allowing an unlimited number of attempts that, therefore, eventually had to end in a win.

EXPERIMENT 1

Method

Participants

Thirty-five participants (20 female participants and 15 male participants; mean age = 20.0 years) were recruited from the Washington University Department of Psychological and Brain Sciences Human Subjects Pool. Participants were tested individually in a small room with a computer and received course credit for their participation.

Materials and procedure

Participants were instructed that they would be making a series of choices between two hypothetical monetary alternatives: Receiving a reward immediately and playing a game in order to receive a larger reward at a later time. The game involved repeated gambles to win the larger reward, with a constant probability of winning on each gamble. If the game alternative was chosen, then the first gamble would take place the next day. If the participant did not win on the first gamble, hypothetically, she would play again 1 week later and once a week thereafter until she did win, at which point the game would end. Therefore, participants were guaranteed to win the larger reward eventually, but the delay until they would win was probabilistic. For each choice, the participant indicated the alternative she preferred, but did not actually play the game or receive the winnings.

In the instructions, the following example was provided:

Which would you prefer?

(1) $25 right now, or

(2) play the following game:

Tomorrow, when you turn on your computer a wheel of fortune will appear, 25% of the wheel will be red and 75% of the wheel will be black. The wheel will be spun. If it lands on red, $50 will be immediately deposited into your bank account. If the wheel lands on black, you win nothing but it will be spun again 1 week from today, and each week thereafter until it finally lands on red, at which time $50 will be deposited into your account, and the game ends.

In this example, participants could either receive $25 immediately or play the game for a 25% chance of winning $50 tomorrow. If they chose to play the game and did not win the $50 on the first gamble, they would play again in 1 week. Participants hypothetically would continue to play each week until they did win the $50. They could not win more than once; after winning the $50, the game would end.

In the experiment proper and to make differences between conditions easier to distinguish, participants were presented on the computer monitor with a simplified format of the choice alternatives on each condition. For example, participants would see the following choices:

$25 with a 100% chance immediately or $50 with a 25% chance tomorrow and once a week until you win

Note that the two formats of the choices (i.e., those presented in the instructions and those in the experiment) are identical, and that the simplified format contains all of the essential information participants needed to make their choice. The simplified format made it easier for participants to recognize when aspects of the choice alternatives changed (i.e., the probability or the amounts of money). Participants could view the instructions (and, therefore, an example of the full format of the game) at any time during the experimental trials. Before the experiment began, participants were given three practice trials. The values used for the practice trials were similar, but not identical, to those used in the experimental trials.

<table>
<thead>
<tr>
<th>String</th>
<th>Delay ($D$)</th>
<th>Amount ($A$)</th>
<th>Probability ($p$)</th>
<th>Expected value ($EV = p \times A$)</th>
<th>Subjective value ($SV = EV / (1 + D)$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>0</td>
<td>1</td>
<td>.5000</td>
<td>.5000</td>
<td>.5000</td>
</tr>
<tr>
<td>LW</td>
<td>1</td>
<td>0</td>
<td>.2500</td>
<td>-.1250</td>
<td>.0000</td>
</tr>
<tr>
<td>LLL</td>
<td>2</td>
<td>-1</td>
<td>.1250</td>
<td>-.1250</td>
<td>.0417</td>
</tr>
<tr>
<td>LLLL</td>
<td>3</td>
<td>-2</td>
<td>.0625</td>
<td>-.1250</td>
<td>-.0313</td>
</tr>
<tr>
<td>LLLLL</td>
<td>4</td>
<td>-3</td>
<td>.0313</td>
<td>-.0938</td>
<td>-.0188</td>
</tr>
<tr>
<td>***</td>
<td>5</td>
<td>-4</td>
<td>.0156</td>
<td>-.0625</td>
<td>-.0104</td>
</tr>
<tr>
<td>***</td>
<td>6</td>
<td>-5</td>
<td>.0078</td>
<td>-.0391</td>
<td>-.0056</td>
</tr>
<tr>
<td>***</td>
<td>7</td>
<td>-6</td>
<td>.0039</td>
<td>-.0234</td>
<td>-.0029</td>
</tr>
<tr>
<td>***</td>
<td>8</td>
<td>-7</td>
<td>.0020</td>
<td>-.0137</td>
<td>-.0015</td>
</tr>
<tr>
<td>***</td>
<td>9</td>
<td>-8</td>
<td>.0010</td>
<td>-.0078</td>
<td>-.0008</td>
</tr>
<tr>
<td>***</td>
<td>10</td>
<td>-9</td>
<td>.0005</td>
<td>-.0044</td>
<td>-.0004</td>
</tr>
</tbody>
</table>

Sum     |              |             |                  |                               | .0054                               |

Note: SV, subjective value; EV, expected value; W refers to a string with an immediate win, LW refers to a string with a loss followed by a win, LLW refers to a string with two losses followed by a win, etc. Each gamble consists of a $1 cost with the chance to win $2. Delay ($D$) refers to the length of the string (i.e., number of losses before a win); Amount ($A$) refers to the sum of the amount won ($2) minus the amount lost in the string ($1 for each gamble); and Probability ($p$) refers to the probability of that string of gambles occurring.

Table 1. Calculation of $SV$ of the 10 most probable strings of an even-money $1 gamble
A two (amount) × six (probability) within-subjects design was used for a total of 12 conditions. Two amounts ($50 and $2000) of the larger reward in the game alternative were used in order to determine if there was an effect of amount on the discounting rate of repeated gambles. Six probabilities of winning the larger amount in the game alternative were used (80%, 60%, 40%, 25%, 10%, and 5%). These probabilities were the chance of winning the larger reward on each gamble if the participant chose to play the game.

In order to determine the subjective value of a repeated gamble, the amount of the smaller, immediate reward was titrated six times in each amount–probability condition. On the first trial in a condition, the amount of the smaller, immediate reward was half the amount of the larger, probabilistic reward in the game alternative (i.e., $25 in the $50 conditions and $1000 in the $2000 conditions). For example, the first trial in one repeated gambles condition might be between receiving $1000 immediately and playing the game to win $2000. For each subsequent trial in the condition, the amount of the smaller reward was adjusted based on the participant’s previous choice (Du, Green, & Myerson, 2002). The size of each adjustment was half that of the preceding adjustment. If in the previous example the participant chose the immediate $1000, then the amount of this reward would be decreased to $500 on the next trial. If the participant instead chose the $2000 of the game alternative, then the amount of the smaller reward would be increased to $1500. If on the second trial, the participant then chose the $1500, the amount of the smaller reward on the third trial would be decreased to $1250 (half of the previous adjustment). After six trials, this titrating procedure converged on an immediate, certain amount of money that was approximately subjectively equivalent to the larger reward in the game alternative (i.e., the subjective value of the repeated gamble). Subjective value was estimated as the amount of the smaller reward that would have been presented had there been an additional (seventh) trial (Du et al., 2002). Half of the participants completed all of the repeated gambles conditions with the $50 reward first, followed by all of the conditions with the $2000 reward; the other half completed the $2000 reward conditions before the $50 reward conditions. For each reward amount, the six probability conditions were administered in a random order.

Results

Figure 1 shows the mean relative subjective value for both the $50 and $2000 rewards as a function of the odds against receiving the reward on each gamble. Equation (1) was fit to the indifference points. The curves in figure 1 represent the hyperboloid function (Equation (1)) that best fit the data as a function of odds against. Notice that in figure 1, relative subjective value (i.e., subjective value as a proportion of the actual amount of the larger reward) is used as the dependent variable in order to facilitate comparison between the two reward amounts. As may be seen, the hyperboloid function provided very good fits to the mean for both the $50 ($R^2 = .978$) and $2000 ($R^2 = .980$) rewards. In addition, we tested the significance of the regression coefficient estimate of $s$ when fit to the mean indifference points for each amount separately. In both cases, the exponent, $s$, was significantly less than 1.0 (both $p < .001$). This finding is consistent with that obtained in other discounting studies and suggests that a simple hyperbola in which $s$ is equal to 1.0 is insufficient to account for human discounting decision making (Green et al., 1999a, 2014).

Although small, there was a reliable difference between the relative subjective values of the small and large rewards at each of the six odds against tested: The $50 reward was discounted more steeply (i.e., a lower relative subjective value) than the $2000 reward at each odds against. Although we examined discounting as a function of odds-against receipt, the direction of this effect of amount is consistent with those observed in delay discounting research (in which larger amounts are discounted less steeply), rather than those observed in probability discounting research (in which larger amounts are discounted more steeply).

To determine whether the amount effect was statistically significant, we first computed the areas under the empirical discounting curves for each participant (Myerson, Green, & Warusawitharana, 2001). Area under the curve (AuC) provides a theoretically neutral measure of the degree of discounting because it does not assume a particular mathematical form of the discounting function. In addition, because we are testing a two-parameter hyperboloid function, the estimate of the discounting parameter, $b$, will be affected by the estimate of the other parameter, $s$. Thus, comparing $b$ estimates would result in a biased estimate of the differences between reward amounts. In order to calculate AuC, indifference points are converted into a proportion of the undiscounted, larger amount of reward (i.e., $50 or $2000), and each odds-against receipt is converted into proportions.
of the largest odds against examined (i.e., 19 for the 5% probability). This procedure normalizes all values so that regardless of the undiscounted amount ($A$ in Equation (1)), AuC ranges from 0.0 (indicating maximal discounting) to 1.0 (indicating no discounting). A paired samples $t$-test comparing AuCs for the $50$ and $2000$ conditions revealed no significant difference in the degree of discounting between reward amounts, $t(34) = 1.36, p = .182$.

Finally, we tested the hypothesis that the subjective value of a repeated probabilistic reward could be modeled as a function of the expected delay until that reward is received. For each probability condition, we calculated the average delay an individual would experience before winning, taking into consideration that all of the gambles occurred 1 week apart and that the first gamble of a series occurred the next day. Expected delay was calculated using the method proposed by Rachlin, Logue, Gibbon, and Frankel (1986) using the following equation:

$$D = \frac{t}{p} - t + i$$

(2)

where $D$ is the expected delay, $i$ is the time between gambles (1 week in this case), $p$ is the probability of winning on each gamble, and $i$ is the initial delay before the first gamble (1 day in this case). For purposes of comparison with later experiments, $t$ and $i$ were calculated in terms of months (i.e., a delay of 1 week = .25 months and a delay of 1 day = 1/30 months). For simplification, Equation (2) can be rewritten in terms of odds-against receipt, $\theta$ (Rachlin & Siegel, 1994):

$$D = \theta t + i$$

Table 2 shows the expected delays (in months) to a win as a function of each probability when the delay between gambles is 1 week and there is an initial 1-day delay before the first gamble. The mean subjective values were fit to Equation (1), but with $X$ representing expected delay. When fit as a function of expected delay, the hyperboloid function provided very good fits for both the $50$ ($R^2 = .981$) and $2000$ ($R^2 = .983$) rewards, and the exponent $s$ was significantly less than 1.0 in both cases ($ps < .001$). Because $D$ is a linear function of $\theta$, the fits and significance levels of the hyperboloid function to expected delay are similar to the fits to odds against.

Table 2. Expected delays (in months) until a win during repeated gambles in Experiment 1

<table>
<thead>
<tr>
<th>Probability of receipt</th>
<th>Expected delay (months)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.0333</td>
</tr>
<tr>
<td>0.8</td>
<td>0.0958</td>
</tr>
<tr>
<td>0.6</td>
<td>0.2000</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4083</td>
</tr>
<tr>
<td>0.25</td>
<td>0.7833</td>
</tr>
<tr>
<td>0.1</td>
<td>2.2833</td>
</tr>
<tr>
<td>0.05</td>
<td>4.7833</td>
</tr>
</tbody>
</table>

*Note: Expected delays were estimated from Equation (2) using a delay of 1 week between gambles, an initial 1-day delay before the first gamble, and the probability of receipt.*

Discussion

The results of Experiment 1 suggest that when there are multiple opportunities over time to win a probabilistic reward, subjective value may be a function of the expected delay until receipt of the reward rather than the probability of receipt on each gamble. We observed a consistent, albeit statistically nonsignificant, difference in the degree of discounting between the two amounts that was in the direction typically observed with delayed rather than probabilistic rewards. Although a null effect of amount stands in contrast to the robust effects of amount found with typical delay discounting and probability discounting tasks, we would note that out of the 210 comparisons (35 participants × 6 probabilities), the delay amount effect was the most common finding: The smaller amount was discounted more steeply 122 times (58%), the larger amount was discounted more steeply 63 times (30%), and the two rewards were discounted equally only 25 times (12%). Moreover, the current experiment examined only one interval between gambles. We suspect that with a longer interval between gambles, the amount effect would be more evident (as, indeed, we find in the next experiment).

Previous work has tested the effect of intertrial interval on choices in which multiple gambles were played, but in which probabilities changed on each gamble (Rachlin et al., 1986; Rachlin & Siegel, 1994; Silberberg, Murray, Christensen, & Asano, 1988). In general, as the interval between gambles increased, choice of the more risky outcome decreased, although Silberberg et al. also found no difference when groups were informed of the number of trials they would be playing. In each of these prior experiments, probabilities in the gamble alternative changed from trial to trial. Because the same gamble was not played repeatedly, it is likely that participants’ choices would not have been affected by how many times they would have to play that particular gamble on average before they would win.

Experiment 2 examined the effect of the interval of time between gambles using a procedure like that of Experiment 1. We also determined whether an amount effect would be observed across the different intervals and whether repeated gambles still could be modeled as a function of expected delay when there are multiple interval lengths. Importantly, we also compared our results with one-shot probability discounting tasks with equivalent probabilities and with delay discounting tasks with similar delays to those of the expected delays in our repeated gambles task.

EXPERIMENT 2

Method

Participants

Eighty participants (43 female participants, 36 male participants, and 1 not reported; mean age = 20.4 years) were recruited from the Washington University Department of Psychological and Brain Sciences Human Subjects Pool. Participants were tested individually in a small room with a computer and received either course credit or nominal payment for their participation. All participants were treated...
identically regardless of how they were compensated, and performance in the experiment was not tied to the amount of compensation.

**Materials and procedure**

The procedure was similar to that used in Experiment 1, with the primary differences being the manipulation of the interval between gambles in the game alternative and the introduction of additional comparison conditions. Participants were divided into two groups. In Group 2A (40 participants; mean age = 19.6 years), participants completed a task similar to that in Experiment 1, except that the interval between gambles in the game alternative was 1 and 6 months. For example, participants might be given the following choices:

- $25 with a 100% chance immediately or $50 with a 25% chance tomorrow and once a month until you win

A two (interval) × two (amount) × six (probability) within-subjects design was used in this group. The same larger amounts ($50 and $2000) and probabilities (80%, 60%, 40%, 25%, 10%, and 5%) as in Experiment 1 were used. Participants completed all of the conditions at a single interval (1 or 6 months) before completing all of the conditions at the other interval. Within each interval condition, participants completed all of the probability conditions with a single reward amount (i.e., $50 or $2000) before completing all of the conditions at the other amount. The order in which all interval and amount conditions were presented had an effect across participants, and for each interval–amount condition, the probability conditions were administered in a random order. A titrating procedure identical to that used in Experiment 1 was used to estimate the subjective value of the larger reward in the game alternative.

Participants in Group 2B (40 participants; mean age = 21.3 years) completed a simple probability discounting task and a simple delay discounting task, each at two reward amounts ($50 and $2000) and probabilities (80%, 60%, 40%, 25%, 10%, and 5%) within-subjects design was used for a total of 12 conditions. In the simple delay discounting task, participants made a series of choices between receiving an amount of money immediately (e.g., $25 right now) and receiving a larger amount of money at a later time (e.g., $50 in 3 years). A two (amount: $50 and $2000) × seven (delay: 1 week, 1 month, 3 months, 6 months, 1 year, 3 years, and 10 years) within-subjects design was used for a total of 14 conditions. These delays were selected because they span the range of expected delays used in Experiment 1 and in Group 2A.

Participants in Group 2B completed both the simple probability and the simple delay tasks. Within each task, participants completed all of the conditions at one amount (i.e., $50 or $2000) before completing all of the conditions at the other amount. The order in which task and amount conditions were presented was counterbalanced across participants. For each task–amount condition, the probability or delay conditions were administered in a random order. For each condition, the same titrating procedure as in Experiment 1 was used to estimate the subjective values of the larger, probabilistic or delayed rewards.

**Results**

Figure 2 shows the mean relative subjective values for the $50 and $2000 rewards as a function of the odds-against receipt at the 1- and 6-months intervals for Group 2A. The 1-week results of Experiment 1 are shown for comparison. The curves represent the hyperboloid function (Equation (1)) that best fit the data. As may be seen, the hyperboloid model provided excellent fits to the mean for all four conditions (all $R^2$s > .98; Table 3), and the exponent, s, was significantly less than 1.0 in all four cases (all ps < .001).

To test if there was a difference in the degree of discounting between the 1- and 6-months conditions, we compared the

4In order to determine whether the order in which the amount conditions were presented had an effect, we added order as an additional between-groups variable in our analyses of variance (ANOVAs). We combined participants across Experiments 1 and 2 because of the similarity of experimental design, thereby reducing the number of statistical tests to conduct and, most importantly, to increase the power for finding an effect if there were one. We conducted a three (interval) × two (amount) × two (order) mixed-groups ANOVA. There was no overall effect of order ($F(1, 69) < 1.0, p = .983), and no interaction with amount ($F(1, 69) < 1.0, p = .48) or with interval ($F(2, 69) = 1.605, p = .208). We, thus, can rule out an order effect in Experiments 1 and 2. In Experiments 3A and 3B, there were far more conditions, and, therefore, the likelihood of finding an order effect (i.e., power) would be low because of the small number of participants within a given order. As a consequence, in light of the fact that there was no effect of order in Experiments 1 and 2 and the fact that the order in which conditions were studied was fully counterbalanced in all the experiments, we do not present such analyses for Experiments 3A and 3B.
The effect of amount in the 1- and 6-month conditions was in the direction opposite to that typically observed with probability discounting: The $50 reward was discounted significantly more steeply than the $2000 reward, $F(1, 39) = 18.06, \, p < .001, \, \eta^2_p = .316$. Figure 3 shows that this effect of amount was consistent across both the 1- and 6-months interval conditions, as well as the 1-week interval condition from Experiment 1. There was no significant interaction between interval and amount ($p = .14$), suggesting that the effects of amount and interval were similar across conditions.

The mean relative subjective value and best-fitting hyperboloid functions from the 1-week condition from Experiment 1 were replotted in figure 2 in order to compare the degree of discounting across the three interval conditions studied. Figure 2 shows a consistent effect of interval: As the length of the interval between gambles increased, the degree of discounting increased (i.e., relative subjective value decreased). In order to determine if there was a significant difference between the three interval conditions, we examined data points only from the first interval condition that participants in Group 2A completed. Recall that participants completed all conditions at a single interval (i.e., 1 or 6 months) before completing all conditions at the other interval. We created two distinct groups, each with 20 participants, by dividing Group 2A into those who completed the 1-month conditions first and those who completed the 6-months conditions first. We then only included data from those conditions, in addition to the 1-week condition from Experiment 1 (36 participants), in our analysis. A three (interval)×two (amount) mixed-groups ANOVA confirmed an overall significant effect of interval, $F(2, 72) = 18.72, \, p < .001, \, \eta^2_p = .342$. Bonferroni-corrected post hoc comparisons revealed a significant difference between the 6-months interval and both the 1-week ($p < .001$) and 1-month ($p = .002$) intervals, but no difference between the 1-week and 1-month intervals ($p = .129$).

In order to compare choice of repeated gambles to that of one-shot gambles, we computed the AuC for the simple probability conditions from Group 2B for both amounts. A paired samples $t$-test revealed a significant effect of amount that is consistent with that found typically in probability discounting studies: The $50 reward was discounted less steeply than the $2000 reward, $t(39) = 5.86, \, p < .001$. As can be seen in figure 3, amount affected the degree of discounting differently for the one-shot and the repeated gambles: In the simple probability condition (one-shot gamble), the smaller reward was discounted less steeply (had a greater AuC) than the larger reward, but in all three repeated gambles conditions, the smaller reward was discounted more steeply than the larger reward.

We next evaluated whether the discounting of repeated probabilistic rewards could be modeled as a function of delay. We converted probabilities to expected delays using Equation (2), taking into account the interval between games (1 or 6 months) and that there was an initial 1-day delay until the first game could be played. Delays were converted into months for both conditions. Equation (1) then was fit as a function of the respective expected delays for each of the two reward amounts and for each of the two interval conditions separately. The hyperboloid model provided excellent fits as a function of expected delay for both the 1- and 6-months conditions, at each reward amount ($\text{all } R^2_s > .98$; Table 3), and the exponent, $s$, was significantly less than 1.0 in all four cases ($\text{all } ps < .001$).

Finally, we tested whether a single hyperboloid function could account for choice across all three interval conditions. Figure 4 presents the indifference points for each interval condition and the best-fitting hyperboloid as a function of expected delay (solid curves) for the $50 (top panel) and $2000 (bottom panel) reward conditions. Despite collapsing data across different conditions and from the 1-week condition of Experiment 1 that included a different set of participants, the hyperboloid function provided a very good description of choice for both the $50 (R^2 = .949)$ and $2000 (R^2 = .967)$ conditions, and the exponent, $s$, was significantly less than 1.0 because the .0333 (1-day) delay is included in all three interval conditions and because in each condition the indifference point was the same (1.0), we removed the two redundant cases so as not to artificially inflate the fits of the hyperboloid.

Table 3. Variance accounted for ($R^2$) by Equation (1) as a function of odds against and of expected delay in Experiments 1 and 2

<table>
<thead>
<tr>
<th></th>
<th>Odds against (probability)</th>
<th>Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>$50$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 week</td>
<td>.978</td>
<td>.981</td>
</tr>
<tr>
<td>1 month</td>
<td>.984</td>
<td>.984</td>
</tr>
<tr>
<td>6 months</td>
<td>.993</td>
<td>.993</td>
</tr>
<tr>
<td>$2000$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 week</td>
<td>.980</td>
<td>.983</td>
</tr>
<tr>
<td>1 month</td>
<td>.996</td>
<td>.991</td>
</tr>
<tr>
<td>6 months</td>
<td>.994</td>
<td>.994</td>
</tr>
</tbody>
</table>

Note: Data are from the 1-week condition of Experiment 1, and the 1- and 6-months conditions of Group 2A.

Figure 3. Mean area under the curve for the $50 (dark bars) and $2000 (light bars) rewards for the 1-week condition of Experiment 1, the 1- and 6-month interval conditions of Group 2A, and the simple probability condition of Group 2B. Error bars represent ±1 standard error of the mean.
Discussion

The results of Experiment 2 are consistent with those of Experiment 1 in that the subjective value of a repeated probabilistic reward can be modeled as a function of the expected delay until its receipt. As can be seen in Figure 3, it also is clear that rewards in the standard (one-shot) probability condition were discounted much more steeply than those in the repeated gambles conditions. This finding is consistent with that found by Keren and Wagenaar (1987) who observed a greater number of participants choosing the riskier option (i.e., they discounted the larger, probabilistic outcome less) when gambles were repeated 10 times than when a single gamble was offered. The current experiment also found that the length of the interval between gambles had a significant effect on choice, such that subjective value decreased as the interval between gambles increased, a finding consistent with earlier work that examined choices in which the probability changed on each gamble (Rachlin et al., 1986; Rachlin & Siegel, 1994; Silberberg et al., 1988). Despite the differences in the degree of discounting across interval conditions, a consistent amount effect was evident in both the 1- and 6-months conditions, such that the smaller reward was discounted more steeply than the larger reward. This latter finding is consistent with the direction of the results obtained in Experiment 1 and lends support to the hypothesis that the subjective value of repeated probabilistic rewards is controlled by the expected delay until their receipt.

It may be of interest to note that there were two cases in which the expected delay was identical for different interval–probability combinations. A 6-months–40% repeated gamble and a 1-month–10% repeated gamble both have an expected delay of about 9 months; a 6 months–80% repeated gamble and a 1 month–40% repeated gamble both have an expected delay of about a month and a half. Interestingly, in both cases, rewards in the 6-month interval condition were discounted less steeply than those in the 1-month interval condition (figure 4). It makes little sense that rewards at a 6-month interval would be discounted less steeply than those at a 1-month interval, so rather it is likely that the lower probabilities in these 1-month conditions (10% and 40% vs. 40% and 80% in the 1- and 6-month conditions, respectively) had a greater effect on choice. That is, the difference between a 10% and 40% chance of receipt has greater influence on choice than the difference between repeated delays of 1 and 6 months (e.g., Vanderveldt, Green, & Myerson, 2015).

It should be noted, however, that expected delay was estimated using a simplified approach in which fixed intervals were assumed. Different estimates would be obtained had we used variable intervals between rewards (e.g., Mazur, 1984). Even if expected delays are equivalent, lower-probable outcomes have greater variability with regard to the number of attempts before a win (i.e., the variance of a geometric distribution increases as the probability of a win decreases). Therefore, the alternative with the lower probability, but shorter delay (i.e., the 1 month–10% and the 1 month–40% repeated gambles) might be discounted more steeply than the alternative with the higher probability, but longer delay (i.e., the 6 months–40% and the 6 months–80% repeated gambles) because participants are averse to the greater variability associated with the lower-probable alternative (but see Kacelnik & Bateson, 1996).

In Experiments 1 and 2, participants were told that if they chose the game alternative, then they would continue to play until eventually they won the reward. Thus, the reward was certain, but the delay until receiving that reward was
probabilistic. The question remains as to whether our findings would be observed in a situation in which there was a fixed number of gambles to play. Rather than playing the game until a win occurred, a participant would be permitted, for example, only 10 gambles (each separated by an interval of time). Would choice in a repeated gambles situation still be a function of expected delay when the number of gambles permitted was limited?

Previous work has shown that choice can differ depending on whether there is one gamble or a fixed, larger number of gambles to win a reward (Coombs & Bowen, 1971; Keren & Wagenaar, 1987). Much of the prior research has examined gambles in which a one-shot gamble was divided into repeated, equal-sized smaller gambles, such that the sum of the smaller repeated gambles had the same expected value as the one-shot gamble (e.g., a 25% chance of winning $100 vs. a 25% chance of winning $1 played 100 times). Generally, more participants chose the outcome with the higher expected value (i.e., they made more rational choices) when the gamble alternative was a repeated gamble than when it was a one-shot gamble. Because the repeated and the one-shot gambles in each of these prior experiments were equated on expected value, choice of one-shot and repeated gambles of the same amount could not be compared.

Recognizing that amount may affect choice beyond the repeated gambles manipulation, Keren and Wagenaar (1987) conducted a follow-up experiment in which the amounts of the outcomes in the repeated gambles condition were the same as those in the one-shot gamble condition. Again, they found that participants were more likely to choose the probabilistic outcome in the repeated gambles condition than in the one-shot condition. It is to be noted, however, that their participants were allowed to win more than once in the repeated gambles condition, thereby allowing for the possibility of winning much more in the repeated gambles condition than in the one-shot condition. Experiment 3 permitted only one win in a repeated gambles condition, thus equating the amount won across conditions and allowing for a controlled investigation of choice in one-shot and repeated gambles. Furthermore, repeated gambles in the Keren and Wagenaar experiment were played in rapid succession, and so, the effect of delay until a win could not be assessed. Experiment 3 created an interval between gambles; this may more closely match probabilistic choices in naturalistic settings and allows an assessment of the effects of both probability and delay on choice in limited repeated gambles.

Experiments 3A and 3B were designed to extend the findings from Experiments 1 and 2 to situations involving a fixed number of attempts at winning a reward. With a one-shot simple probability discounting task (Group 2B), we obtained the typical reverse amount effect in which a larger probabilistic reward is discounted more steeply than a smaller one. When the gamble could be repeated indefinitely until a win (Experiment 1 and Group 2A), the larger reward was discounted less steeply than the smaller one (the typical amount effect observed with delayed rewards). Experiments 3A and 3B varied the number of gambles permitted, allowing for a determination of the point at which people respond to a repeated gamble as if it were a guaranteed win, in which case, it is only a matter of the delay until receiving the reward.

**EXPERIMENT 3A**

In the previous two experiments, there was an initial 1-day delay until the first gamble could be played in the game alternative. Every gamble after that occurred after a longer, fixed delay (i.e., 1 week, 1 month, or 6 months). The initial 1-day delay may unnecessarily complicate the procedure and analyses; therefore, before conducting Experiment 3, we determined whether this 1-day delay markedly affected the degree of discounting. Thirty-six participants (23 female participants and 13 male participants; mean age = 21.6 years) were studied on the same condition as those in Group 2A from Experiment 2 except that they were told that the first gamble would take place immediately; if they did not win on that gamble, it would be repeated in 1 month (6 months) and once a month (once every 6 months) until they did win. For example, participants might be given the following choices:

- $25 with a 100% chance immediately or $50 with a 25% chance immediately and once a month until you win

The intervals between gambles, the amounts of the larger reward, and the probabilities of receiving the larger reward at each gamble were identical to those used in Group 2A.

To determine if the 1-day delay to the first gamble in the game alternative significantly affected choice, we compared the AuCs of this group with those of Group 2A from Experiment 2. A two (group) x two (interval) x two (amount) mixed-groups ANOVA revealed no overall difference between groups ($p = .376$) and no interaction of group with interval or with amount (both $ps > .55$). These findings suggest that the 1-day delay did not significantly affect the subjective value of the larger rewards in the game. In Experiments 3A and 3B, therefore, we simplified the procedure by removing the 1-day delay so that if the game alternative was chosen, the first gamble was played immediately.

**Method**

**Participants**

One hundred and forty-seven participants (88 female participants and 59 male participants; mean age = 19.9 years) were recruited from the Washington University Department of Psychological and Brain Sciences Human Subjects Pool. Participants were tested individually in a small room with a computer and received course credit for their participation.

**Materials and procedure**

The procedure used for this experiment was similar to that of Experiments 1 and 2, with the primary difference being the manipulation of the number of gambles permitted in the
game alternative. In Experiments 1 and 2, participants played the gamble repeatedly until they won. As a consequence, the reward was guaranteed, but the actual delay until they would win and receive the reward was uncertain. In the current experiment, the number of gambles permitted was limited, but as in the previous experiments, the game would stop after a win. If participants did not win after the specified number of gambles, then they would not win any money.

In the instructions, participants were shown the following example:

Which would you prefer?
(1) $25 right now,
or
(2) play the following game:

A wheel will appear on the computer screen. 25% of the wheel will be red and 75% of the wheel will be black. The wheel will be spun right now. If the spinner lands on red, you’ll immediately receive $50. If the spinner lands on black, you win nothing but the wheel will be spun again 1 month from today, and once every month thereafter. This will go on for a maximum of 5 months for a total of 6 tries (1 immediate try and once a month thereafter for 5 months). If you win (i.e., the spinner lands on red) before the 6th try, the game will end and you will receive $50. If you have not won after 6 tries, the game will end and you will receive nothing.

In this example, if participants chose to play the game, they would play one time immediately, and then each month for five additional months. Hypothetically, they would continue to play until they won or until they completed all six gambles (i.e., trials). They could not win more than once; if they chose to play the game in this example, $50 was the most they could win.

In the experiment proper, participants were shown a simplified format of the question in order to facilitate comprehension:

$25 with a 100% chance immediately or $50 with a 25% chance immediately and once a month for a maximum of 6 tries

Before the experiment, participants were given four practice questions, which included one amount, two different numbers of gambles (i.e., trials), and two probabilities. The values used were similar, but not identical, to those used in the experimental conditions. Following the practice questions, participants took a short quiz to test their understanding of the procedure. Participants who did not answer all questions correctly were asked to re-read the instructions and take the quiz again. If again the participant did not answer all questions correctly (i.e., twice did not pass the quiz), the experimenter went over each answer. Participants who did not pass the quiz after a second attempt were allowed to complete the experiment, but their data were removed before analysis.

Participants were randomly assigned to one of three groups, which differed in the interval between gambles (1 week, 1 month, and 6 months). Within each interval group, a two (amount) × six (gambles) × six (probability) within-subjects design was used. The amounts of the larger reward ($50 and $2000) and the probabilities of winning on each gamble (80%, 60%, 40%, 25%, 10%, and 5%) were identical to those used previously. The number of gambles (i.e., tries) was 1 (a one-shot gamble), 2, 5, 10, 20, and 100. Within each interval group, participants completed all of the conditions at a single reward amount (i.e., $50 or $2000) before completing all of the conditions at the other reward amount. The order in which amount conditions were presented was counterbalanced across participants. Within each reward amount, the number of gambles was presented randomly, and within each amount–gamble condition, the probability conditions were administered in a random order. A titrating procedure identical to that used in Experiments 1 and 2 was used to estimate the subjective value of the larger reward in the game alternative.

Results

Seven participants were removed because they did not pass the instructions quiz after two attempts. The total number of participants included in the following analyses is 140 (46 participants in the 1-week interval condition, 48 participants in the 1-month interval condition, and 46 participants in the 6-months interval condition; mean age = 19.9 years). Equation (1) was fit to the mean indifference points as a function of odds-against receipt for each interval–amount–gamble conditions individually, for a total of 36 fits. The $R^2$s ranged from .932 to .999, with a median fit of .989, and the exponent, θ, was significantly less than 1.0 in all cases (all ps < .02).

Figure 5 shows the mean AuC for each of the 36 interval–amount–gamble conditions. The top, middle, and bottom panels show the AuC for the 1-week, 1-month, and 6-month interval conditions, respectively. A three (interval) × two (amount) × six (gambles) mixed-groups ANOVA revealed an overall significant effect of amount such that the $2000 was discounted more steeply than the $50, $F(1, 137) = 66.947, p < .001, ηp^2 = .328$. There was a main effect of number of gambles ($F(5, 133) = 112.091, p < .001, ηp^2 = .81)$, and Bonferroni-corrected pairwise comparisons revealed that all gambles conditions were significantly different from each other (all ps < .001). There was a significant amount × gambles interaction, $F(5, 133) = 3.264, p = .008$, $ηp^2 = .11$. The $50 and $2000 rewards were discounted at significantly different rates in the 1, 2, 5, 10, and 20 gambles conditions (all $Fs(1, 137) > 17.93, \text{all } ps < .001, \text{all } ηp^2 s > .11$), but the two rewards were not discounted at different rates in the 100 gambles condition ($p = .135$). Furthermore, the size of the difference in AuC between amounts decreased as the number of gambles increased, with the two amounts discounted approximately the same in the 100-gambles condition.

There was no main effect of interval or an interval × amount interaction (both $ps > .51$), but there was a significant interaction between interval and number of gambles, $F(10, 268) = 3.028,$
clear switch point, we did observe a change in the effect of amount across gambles conditions. As the number of gambles increased from 1 to 100, the difference in degree of discounting between the two amounts became smaller, and at the 100-gambles condition, the degree of discounting between the two amounts was not significantly different. This pattern of results suggests that with a larger number of gambles, the degree of discounting of the two amounts might eventually reverse.

We also did not observe a main effect of interval, a finding inconsistent with Experiment 2 in which the degree of discounting increased as the interval between gambles increased. However, there was a significant interaction between the number of gambles and interval: At the 100-gambles condition only, there was a linear effect of interval such that as the interval length increased, the degree of discounting increased, and this effect was not found at any other gambles conditions. It is possible that an effect of interval only emerges when the number of gambles is very large. This finding would help explain why an effect of interval was observed in Experiment 2, in which an unlimited number of gambles was permitted.

The range of the number of gambles used in the current experiment may have been insufficient to observe an effect of interval and to observe a switch point in the effect of amount. To address this possibility, we increased the number of gambles to above 100. In addition, we included an unlimited gamble condition in order to observe the entire spectrum of the number of gambles, from one to unlimited, within the same individual.

**EXPERIMENT 3B**

**Method**

**Participants**

One hundred participants (63 female participants, 35 male participants, and 2 not reported; mean age = 21.4 years) were recruited from the Washington University Department of Psychological and Brain Sciences Human Subjects Pool. Participants were tested individually in a small room with a computer and received either course credit or nominal payment for their participation. All participants were treated identically regardless of how they were compensated, and performance in the experiment was not tied to the amount of compensation.

**Materials and procedure**

The procedure was the same as that used in Experiment 3A, with a few modifications. Participants were randomly assigned to one of two groups that differed in the interval between gambles (1 week or 2 months). The number of gambles (tries) was 1, 10, 100, 500, and an unlimited number of gambles. The unlimited gamble condition was similar to that used in Experiments 1 and 2. All other procedures were identical to those used in Experiment 3A. Therefore, within each
interval group, a two (amount) × five (gambles) × six (probability) within-subjects design was used.

Results
All participants passed the instructions quiz and were included in the analyses. There were 50 participants in the 1-week interval condition and 50 participants in the 2-months interval condition.

Equation (1) was fit to the mean indifference points as a function of odds against for each interval–amount–gambles condition individually, for a total of 20 fits. The $R^2$s ranged from .957 to .999, with a median fit of .983, and $s$ was significantly less than 1.0 in all cases (all $p < .001$).

Figure 6 shows the mean AuC for each of the 20 conditions. The top panel shows the AuC for the 1-week condition individually, for a total of 20

![Image](http://example.com/image.jpg)

Figure 6. Mean area under the curve for the $50 (dark bars) and $2000 (light bars) rewards as a function of the number of gambles (i.e., tries) permitted in Experiment 3B. The top panel shows the data for the 1-week interval condition, and the bottom panel shows the data for the 2-month interval condition. Error bars represent ±1 standard error of the mean.

The opposite was found in the unlimited gambles condition: $50 was discounted more steeply than $2000, consistent with the reverse amount effect typically found in probability discounting. The two amounts did not differ significantly in any of the other gambles condition (all $F$s $< 1.498$, all $p > .22$), but there was a clear trend in the relative degree of discounting of the two amounts across the five conditions: As the number of gambles increased, participants switched from discounting the $2000 more to discounting the $50 more. This finding is consistent with our hypothesis that as the number of chances to win a reward increases, people stop considering the probability of winning and begin to consider the expected delay until that win.

No other interactions were significant (all $p > .10$), suggesting that other than the degree of discounting, the two interval conditions produced similar patterns of choice. Although there were no significant interactions with interval, it is worth noting that the size of the amount effect across gambles conditions is larger in the 2-months condition than in the 1-week condition, consistent with the findings from Experiments 1 and 2 in which stronger effects were observed.

Discussion
The results of Experiment 3B support our original hypotheses and are consistent with the direction of the results obtained in Experiment 3A. First, the effect of amount on degree of discounting changed as the number of gambles increased: With a one-shot gamble or when only a few gambles were allowed, the $2000 reward was discounted more steeply than the $50 reward; as the number of gambles allowed increased or was unlimited, the $50 reward was discounted more steeply. This finding suggests that when there are multiple opportunities over time to receive a
probabilistic outcome (i.e., probabilistic waiting), the subjective value of that outcome may be a function of the expected delay until receipt rather than the probability of receipt on each individual gamble.

Second, we found an effect of interval such that as the time between gambles increased, rewards were discounted more steeply (compare the lower panel with the upper panel of figure 6). This finding, along with those from Experiments 1 and 2, is consistent with earlier research suggesting that the interval between gambles plays a significant role in the subjective value of outcomes (Rachlin et al., 1986; Rachlin & Siegel, 1994; Silberberg et al., 1988). Regardless of the interval between gambles, however, similar effects of amount and the number of gambles were observed, suggesting that these results are quite robust.

CONCLUSIONS

Discounting is steeper with smaller amounts of individual delayed rewards than with larger amounts: the amount effect. Discounting is steeper with larger amounts of individual probabilistic rewards than with smaller amounts: the reverse amount effect. The current series of experiments using, as a marker, these opposite effects of reward amount on delay and probability discounting attempted to determine whether probabilistic waiting (a repeated series of probabilistic events separated by fixed intervals ending in a positive outcome) is treated more like probability or more like waiting (delay).

In Experiments 1 and 2, in which participants had an unlimited number of attempts to win a reward, we found an amount effect like that typically found with delay discounting, which led to the conclusion that probabilistic waiting in these experiments is basically waiting—basically a delay effect. But in these experiments, the reward was essentially certain, provided that the participant waited long enough. In terms of the illustration presented in the introduction, if one is willing to wait long enough for a sunny day to go for a drive in the country, a sunny day eventually will come, even if the probability of one is very low on any given day. Figure 4 shows that discounting with probabilistic waiting, across various delay—probability combinations, with different groups of participants, can be almost entirely accounted for by the expected delay to reward.

In Experiment 3, however, where the number of trials was fixed and a series of gambles could end without reward, a more complex picture emerged. With a single probabilistic gamble, a reverse amount effect was found like that typically observed in probability discounting experiments. As the number of gambles (hence, the chance of reward) increased, however, discounting became less probability-like and more delay-like. This transition is shown in figure 6 where for both 1-week and 2-month intertrial intervals, as the number of gambles increased, the reverse amount effect at first diminished and then changed to a direct amount effect. In other words, as an eventual reward became more certain, discounting with probabilistic waiting became more delay-like and less probability-like.

What might this pattern of results say about real-world gambling and gamblers? In choosing between individual small, certain rewards and individual large, probabilistic rewards, people tend to be risk averse (Kahneman & Tversky, 1979). If gambling is viewed solely in terms of individual gambles, then a pathological gambler would be expected to be less risk averse than a non-gambler, or perhaps risk seeking. Holt et al. (2003) found that college students who gambled were indeed less risk averse (i.e., more risk-taking) than were non-gamblers in choosing between individual small, certain rewards and individual large, probabilistic rewards. Studies using one-shot gambling procedures, however, may have limited application to pathological gambling because, by definition, pathological gamblers face repeated, not one-shot, gambles.

Recall that pathological gamblers tend to have steeper delay discount functions than do non-gamblers (e.g., Petry, 2001). Steep delay discount functions (i.e., strong preference for smaller-immediate to larger-delayed rewards) are indicative of impulsiveness and are characteristic of addicts (Bickel & Marsch, 2001; Carroll, Anker, Mach, Newman, & Perry, 2010). But what is more immediate than money you already have? Why should an impulsive person risk losing a sure in-hand sum of money for only a possibility of a later, larger sum? And why should this tendency be greater as delay discounting is steeper? The answer may be that when gambles are repeated many times so that an eventual win is virtually certain, as is the case with pathological gamblers, the gambles are structured in terms of delay—strings of losses followed by a win. Rachlin et al. (2015) showed that in theory, the strings of losses generated by repetition of a gamble, when discounted by delay (proportional to string length), can have positive subjective value even when the expected value of the gamble is negative. Furthermore, the greater the degree of delay discounting, the more positively the gamble is subjectively valued. The present study suggests that people do indeed treat long strings of gambles like they treat delays to a reward, and they do so more as the number of gambles (and the ultimate probability of a win) increases.

It needs to be emphasized nevertheless that the present laboratory experiments differ in several ways from real-world gambling. First, the rewards were hypothetical rather than real. Second, even if real money had been involved, the probabilities were probabilities of winning—there were no costs to playing. Third, the complex atmosphere of casinos or racetracks was not duplicated in the present experiments (Dixon, Jacobs, & Sanders, 2006). Fourth, the participants were college students, and not selected for any particular gambling experience, and well may differ from pathological gamblers (see Welte, Barnes, Tidwell, Hoffman, & Wieczorek, 2014, for a review of gambler demographics). Any of these factors could call into question the results obtained here and the conclusions earlier, and future studies will be needed to adequately address these concerns. On the other hand, the remarkable regularity of the results, their consistency with previous findings, and their conformance with what seems reasonable and intuitively plausible may make them a suitable reference.
point in the development of a comprehensive theory of probabilistic waiting.

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A. Vanderveldt et al.


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