In other work (Moscarini and Postel-Vinay, 2008, 2009a), we find a distinct cyclical pattern of the relative performance of large and small businesses in terms of net job creation. Large employers destroy proportionally more jobs during and right after recessions, and create proportionally more jobs late in expansions, relative to small employers. Differential size growth between small and large firms is strongly positively correlated with the unemployment rate. This pattern is observed both in a 1978-2005 census of U.S. employers, the Business Dynamics Statistics, and among listed companies, in Compustat.

In this paper, we show that this cyclical pattern of relative performance is also reflected in stock returns. Specifically, we show that the difference in returns between benchmark portfolios of small cap stocks and portfolios of large cap stocks is also positively correlated with the unemployment rate. Financial consultants and fund managers commonly recommend investing in small cap stocks during business cycle recoveries. Our findings, while consistent with that advice, pertain to all phases of the business cycle.

We propose an explanation of both facts based on dynamic competition between employers of different sizes and different productivities. The model is a stochastic dynamic version of the job search and wage posting model of Kenneth Burdett and Dale T. Mortensen (1998), which we analyze in detail in Moscarini and Postel-Vinay (2009b). It is a job ladder model, where smaller firms are smaller because they are less productive, offer lower wages, therefore are less attractive to workers and less successful in poaching workers out of competing firms. This lack of competitiveness on the labor market is more of a drawback for small firms during expansions, when all firms want to hire: small firms are more constrained, thus make relatively lower profits, than large firms when unemployment is low. So the return to investing in small vs. large firms should depend positively on the unemployment rate. Simulation of profit rates from the calibrated model confirm this prediction.

The next section presents evidence of the cyclicity of relative stock returns by capitalization. Section II sketches the model and analyzes its prediction on relative stock returns. Section III discusses the results.

### I. Evidence on small cap excess returns

Eugene F. Fama and Kenneth R. French maintain quarterly series of returns (dividend plus capital gains) on various portfolios, including a series of small cap excess returns defined as the difference in average returns between three ‘small’ benchmark portfolios and...
three ‘large’ ones, as described on French’s web site.\textsuperscript{2} We show a plot of that series over 1948:Q1-2009:Q3 in the top panel on Figure 1, together with the detrended civilian unemployment rate, our measure of the economy’s business cycle conditions.\textsuperscript{3} Because the large amount of high-frequency volatility in the excess return series clouds the picture somewhat, we also report, in the bottom panel of Figure 1, the same plot featuring a four-quarter backward moving average of the raw quarterly excess returns.

Apart from some notable off-beat swerves in the late 1960s, late 1970s and around 2000, the excess returns give a clear impression of co-moving with unemployment. The overall correlation between unemployment and both series of excess returns is around 0.15, significant at conventional levels.\textsuperscript{4}

While the Fama and French data consistently covers a very long time period, it has been criticized as potentially suffering biases due to de-listing of some companies. Martijn Cremers, Antti Petajisto and Eric Zitzewitz (2008) have constructed an alternate series of small cap excess returns based on the Russell 2000 index return minus the Standard and Poor’s 500 index return over the period 1979-2008 (available from www.petajisto.net/data.html), which is immune from de-listing bias problems: the average excess return over the sample period is zero. Results based on this alternate source, available on request, are qualitatively the same.

II. The model

We study a stochastic economy where firms commit to employment contracts and workers search randomly for those contracts. Time is discrete. A unit-mass of risk-neutral, infinitely lived workers can be either unemployed or employed at one of a unit mass of risk-neutral firms. Workers and firms maximize payoffs discounted at rate $r$. Firms operate constant-return technologies with heterogeneous productivity levels $\omega$, where $\omega$ is an aggregate component fluctuating randomly between a high and a low state, $\omega_{t}$ and $\omega^{\prime}_{t}$, following a first-order Markov process with transition probabilities $\sigma_{\omega}$ and $\sigma_{\omega}^{\prime}$, and $\theta$ is a fixed, idiosyncratic heterogeneity component, distributed across firms $\theta \sim \Gamma(\gamma)$ with continuous density $\gamma = \Gamma(\gamma)$ over $[\theta, \Gamma(\gamma)]$.

The labor market is affected by search frictions: unemployed workers only meet potential employers sequentially with some probability $\lambda_{t}$ each period, while employed workers search on the job and face a per-period chance of meeting a vacant job of $\lambda_{t}$. For simplicity we assume uniform sampling of firms by workers: upon meeting a vacant job, workers draw the type $\theta$ of the firm posting the vacant job from $\Gamma$. Employed workers also face a per-period probability $\delta_{t}$ of becoming unemployed. Note that all these transition probabilities, although exogenous, may depend on the aggregate state $\omega$.

The timing is as follows. First, production and payments take place in the current state $\omega_{t}$. The new state $\omega_{t+1}$ of aggregate labor productivity is then realized, at which point employed workers have an opportunity to quit to unemployment. Next, jobs are destroyed exogenously with chance $\delta_{t+1}$. Then the remaining employed workers receive an outside offer with chance $\lambda_{t+1}^{\omega_{t+1}}$ and decide whether to accept it or to stay with their current employer. Finally, each previously unemployed worker receives an offer with probability $\lambda_{t+1}^{\omega_{t+1}}$.

Firms set wages by committing to a future path of state-contingent wages subject to an equal treatment constraint, whereby a firm must pay the same wage to all its workers at all dates. Under commitment, such a wage function implies a value $W_{t}$ for any worker to work for any particular firm at date $t$. Workers will then follow a simple reservation value policy: the unemployed will accept any offer that has higher value than unemployment, $W_{t} \geq U_{t}$, where $U_{t}$ is the workers’ common value of unemployment, while a worker employed on a contract with current value $W_{t}$ will accept any offer that exceeds $W_{t}$.

In this context, the equilibrium allocation of labor into firms will be efficient (subject to the search frictions) if and only if the values offered by firms to workers are a strictly increasing function of firm productivity $\theta$ at all dates, i.e. if more productive firms always pay their workers more. Indeed, if such is the case, then workers rank their preferences over firms according to firm productivity at all dates and always

http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/F-L bench_factor.html

\textsuperscript{2} We detrend the monthly civilian unemployment rate series using a Hodrick-Prescott filter with a smoothing parameter of 8,1E6 at monthly frequency. This particular value of the smoothing parameter is motivated in Moscarini and Postel-Vinay (2009a). We then use the de-trended unemployment rate in the last month of each quarter to match with the quarterly Fama and French data.

\textsuperscript{3} Closer examination, however, reveals that the correlation over the period 1979-2009 is higher at 0.266, while the pre-1979 correlation is 0.083 and not statistically significant.
endeavor to move up the productivity ladder, at the speed allowed by search frictions. We will call any equilibrium having that property a Rank-Preserving Equilibrium (RPE).

Let $L_{t-1}(\theta)$ denote employment size of a type-$\theta$ firm at some date $t-1$. In a RPE, invoking a large numbers approximation, employment at a type-$\theta$ firm evolves following:

\[ L_t(\theta) = L_{t-1}(\theta) (1 - \delta^\omega_t) (1 - \lambda_t^\omega_t \Gamma(\theta)) + \lambda_0^\omega_t u_{t-1} + \lambda_t^\omega_t (1 - \delta^\omega_t) \int_0^\theta L_{t-1}(x) \, d\Gamma(x), \]

where $\Gamma := 1 - \Gamma$ and $u_{t-1}$ is the unemployment rate. Given new aggregate state $\omega_t$, of the $L_{t-1}(\theta)$ workers initially employed by this firm, a fraction $(1 - \delta^\omega_t)$ are not separated exogenously into unemployment. Of these survivors, a fraction $\lambda_t^\omega_t \Gamma(\theta)$ receive an offer from a firm of productivity higher than $\theta$. In a RPE, where the value of an offer is always an increasing function of firm type, that offer is more attractive than what the worker would obtain by staying at the type-$\theta$ firm so they accept it and quit. The initially unemployed $u_{t-1}$ find jobs with chance $\lambda_0^\omega_t$. The employed who have not lost their jobs receive an offer with chance $\lambda_t^\omega_t$, and accept it if their current employer has productivity $x < \theta$.

RPE, which have the desirable property of featuring efficient labor reallocation, are generic in this model. In Moscarini and Postel-Vinay (2009b) we establish that, under the sufficient condition that $L(\theta)$ be initially nondecreasing (i.e. if more productive firms start out no smaller), then any symmetric equilibrium of the dynamic value-posting game played by firms is necessarily Rank-Preserving. As a direct consequence of (1) and the fact that $L(\theta)$ is initially non-decreasing, the initial ranking of firms’ relative sizes is maintained along the RPE path. This result has a simple intuitive explanation, which parallels Burdett and Mortensen’s (1998) steady-state model. First, a more generous offer implies a larger firm size, both because is attracts more workers and retains them more easily. Second, a larger firm size is more valuable to a more productive firm, because each worker produces more. A simple monotone comparative statics intuition thus applies: more productive firms offer more, employ more workers, and earn higher profits.

We further show in Moscarini and Postel-Vinay (2009b) that the Rank-Preserving property suffices to uniquely characterize the equilibrium firm policy, i.e. the value $W_t$ transferred by a firm to its workers as a function of the firm’s type $\theta$ conditional on the aggregate state $\omega_t$ and the wage function that implements it. In so doing we also provide a constructive proof of existence of a RPE. We then go on to exploit equation (1) to establish, among other things, that employment at initially large firms is predicted by the model to be more cyclically sensitive than employment at initially smaller firms, as is observed in the data.

Our focus in the present paper is on the excess returns on small cap firms. Consider a panel of firm data generated by the model in which a sample of firms of various types $\theta \sim \Gamma$ are followed over a number of periods $t = 1, \ldots, T$. Next denote the observed realized wage, employment size and stock market value for a type-$\theta$ firm at date $t$ by $w_t(\theta)$, $L_t(\theta)$, and $V_t(\theta)$, respectively. Further denote the prevailing aggregate state at date $t$ by $\omega_t$. The date-$t$ stock market value of a type-$\theta$ firm, $V_t(\theta)$, equals the expected PDV of profits, which solves:

\[ \Pi(\theta, L_t(\theta), \omega_t) := (\omega_t \theta - w_t(\theta)) L_t(\theta) + \frac{1}{1 + \rho} \mathbb{E}_{\omega_{t+1}}[\Pi(\theta, L_{t+1}(\theta), \omega_{t+1}) | \omega_t], \]

where $L_{t+1}(\theta)$ is given by (1) in each possible next-period aggregate state $\omega_{t+1}$. With (2) in hand, we then define the (backward-looking) return on the firm-$\theta$ asset in the following way, which is consistent with the data used in Section I:

\[ R_t(\theta) = \frac{(\omega_{t-1}\theta - w_{t-1}(\theta)) L_{t-1}(\theta)}{V_{t-1}(\theta)} + \frac{V_t(\theta) - V_{t-1}(\theta)}{V_{t-1}(\theta)}. \]

We next set two thresholds $\theta_L \geq \theta_H$, verify that firms with $\theta \geq \theta_H$ are “large caps” in terms of model-based market valuation, and firms with $\theta \leq \theta_L$ are “small caps”, and define the model-based small cap excess return as $\frac{1}{T} \int_{\theta_L}^{\theta_H} R_t(\theta) \, d\Gamma(\theta)$. Figure 2 shows a time series of small cap excess returns obtained from a simulation of the model.$^5$ The time unit is a quarter. The figure also shows a plot of the (rescaled) unemployment rate to materialize the

$^5$The simulation is based on the algorithm and calibration presented in Moscarini and Postel-Vinay (2009b). We refer the reader to that paper for details.
The excess return series clearly co-moves with unemployment.

How is this pattern explained in the model? The gist of the argument is that, while a positive aggregate productivity shock is good news for all firms, it is comparatively less good news for smaller, less productive firms than for larger firms—and conversely for a negative shock to $\omega$. The reason is twofold. First, given fixed relative size, complementarity between the aggregate and idiosyncratic productivity components in the production technology—output equals $\omega L(\theta)$—directly implies that the impact of fluctuations in aggregate productivity $\omega$ on the value of low-$\theta$ firms is naturally dampened relative to high-$\theta$ firms. This "productivity effect" is further reinforced by the fact that firm size $L(\theta)$ is an increasing function of $\theta$ at all dates. The second effect at work is the dynamic size effect identified in the data in Moscarini and Postel-Vinay (2008, 2009a) and analyzed in Moscarini and Postel-Vinay (2008, 2009b): employment at larger, more productive firms is more responsive to business cycle conditions than employment at smaller firms. This arises in the model because of the nature of competition between employers on the labor market: high-productivity firms are higher up on the workers' job ladder, they can afford paying higher wages and are willing to do so, and as such they are in a position to successfully raid lower-productivity firms for workers. This possibility benefits them most in expansions, when all firms want to hire. Low-productivity firms, on the other hand, are not attractive to employed job seekers and have to trawl the unemployment pool for extra labor. But that pool quickly dries out in a boom, further constraining the expansion of low-$\theta$ firms. Both effects combine into small firms benefitting less, in relative terms, from aggregate expansions, and also suffering less from aggregate contractions. In essence, the labor supply that firms face is less elastic when unemployment is low, more so for small firms which cannot poach workers from larger competitors.

III. Discussion

One alternative view of the evidence we present in Section I is that it is caused by monetary policy, which matters through the credit channel. If the Federal Reserve follows a Taylor rule, so essentially moving interest rates against unemployment, and if small firms are more sensitive to changes in credit conditions, for example because they are more credit constrained, then the correlation between unemployment and the relative performance of large and small firms (in terms of employment and profits) is mediated by endogenous monetary policy. Rather than unemployment constraining small firms in booms and helping them in recessions, it is the interest rate rising during booms that tames their growth and stock returns, and this effect is lessened in recessions.

Obviously our model is completely silent on the potential impact of monetary policy through the credit channel. Indeed, our empirical evidence in Moscarini and Postel-Vinay (2009a) on the cyclical employment performance of small and large firms clearly runs counter a credit channel view, and supports the predictions of our model. However, to entertain the hypothesis outlined above, we estimate a structural $VAR$ featuring the small cap excess return, the Federal Funds Rate (FFR), the CPI inflation rate, and the unemployment rate. We identify the $VAR$ by ordering the variables as listed: each variable responds contemporaneously only to its own shock and to shocks to variables listed after. In other words, relative stock returns react to all three shocks on impact because presumably stock prices are always the first to react and incorporate news. Monetary policy comes second, then inflation, and unemployment comes last, not responding on impact to shocks to any other variables, reflecting the presumption that any unemployment surprise is reflected in stock prices, monetary policy reaction, and inflation almost simultaneously.

In order to give this alternative view the best chance of success, we estimate our $VAR$ with two lags on quarterly data over the 1979-2009 period, for two reasons. First, there is wide consensus that the Taylor...
principle characterizes U.S. monetary policy only after 1979, following Paul Volker’s appointment as FED Chairman. Second, as mentioned in footnote 4, the correlation we document is much stronger after 1979.

Figure 3 plots some relevant impulse response functions. Ignoring the noise and lack of precision, the top right panel in Figure 3 shows that indeed the real FFR declines when unemployment rises. Moreover, consistently with the predictions of the credit channel, the relative returns to small cap firms decline when the FFR increases (bottom left panel). Yet, an innovation to unemployment has a positive and lasting impact of the same sign on small cap excess returns (top left panel). So allowing for an endogenous and non-neutral monetary policy response, assuming it exists, would not completely sever the channel we present.

Our estimated impulse responses remain qualitatively similar when using alternative data sets. We estimated the same VAR on monthly data, drawing the small cap excess return either from the same Fama and French source, or from Cremers, Petajisto, Antti and Zitzewitz (2008). The extreme volatility of monthly stock returns only reduces precision. We also estimated the VAR on all available quarterly data (from 1954, the year at which the FFR is first available). The only noticeable change there is that the response of unemployment to an FFR shock tends to be marginally significantly negative, although quantitatively negligible, in the medium run.

REFERENCES


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